Physics Unit 4 Summary

Further Mechanics

Momentum Concepts

- Momentum = mass x velocity

$$p = mv$$

- Momentum is <u>conserved</u> in collisions and explosions

$$total\ momentum\ before = total\ momentum\ after$$

- Elastic collision: Kinetic energy is conserved. E.g. Elastic ball bouncing on the floor
- Inelastic collision: Kinetic energy is not conserved. E.g. Stone falling on the floor

- Newton's Second Law: *"The net (resultant) force on an object is equal to the rate of change of its momentum"*

$$F = \frac{\Delta mv}{\Delta t}$$

$$F = \frac{mv - mu}{t} = m\left(\frac{v-u}{t}\right)$$

$$F = ma$$

- Impulse = change in momentum = force x time

$$Impulse = \Delta mv = F\Delta t$$

- Area under force-time graph gives the impulse (which also is the change in momentum)

Circular Motion

- Angular velocity: Angle covered in the circle in unit time (rad^{-1})

Angular velocity (omega)

$$\omega = \frac{2\pi}{T}$$

Angles in a full circle

Period: Time to complete a full cycle

$$T = \frac{1}{f} \qquad \Longrightarrow \qquad \omega = 2\pi f$$

- Tangential/linear velocity: Distance covered on the circle in unit time (ms^{-1})

$$v = \frac{2\pi r}{T}$$

Circumference of the circle

- Combining the above equations gives:

$$v = \omega r$$

- Centripetal acceleration:

velocity → magnitude: stays constant
velocity → direction: changes

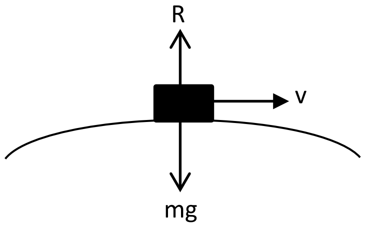
$$a = \frac{v^2}{r} = \omega^2 r$$

- When there's acceleration, there must be a net (resultant) force
  ⇨ Centripetal force, always towards the centre of the circle

- Using $F = ma$ gives:

$$F = \frac{mv^2}{r} = m\omega^2 r$$

When the object is about to lift off or feels weightless, normal reaction force is zero
E.g.



$$\downarrow F = ma$$

$$mg - R = \frac{mv^2}{r}$$

For weightlessness, $R = 0$

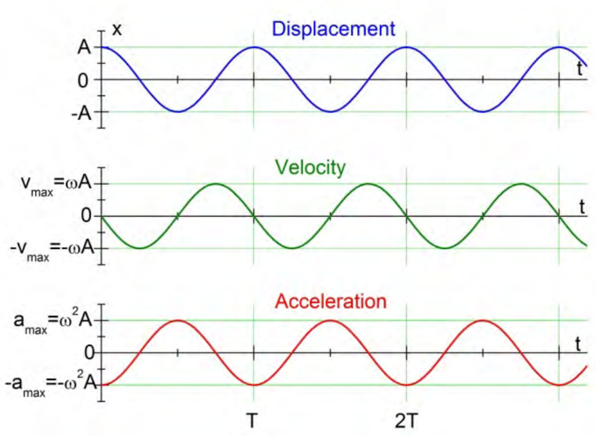
$$mg = \frac{mv^2}{r}$$

Simple Harmonic Motion

- Necessary condition: "*Acceleration is directly proportional to displacement from centre, but in the opposite direction*"

$$a = -(2\pi f)^2 x = -\omega^2 x$$

- Graphs against time:



$$x = A\cos 2\pi f t$$

$$v = \pm 2\pi f\sqrt{A^2 - x^2}$$

$$Maximum\ velocity\ = 2\pi f A$$

(at the centre)

$$Maximum\ acceleration = (2\pi f)^2 A$$

(at maximum displacement)

- Period of a mass-spring system:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

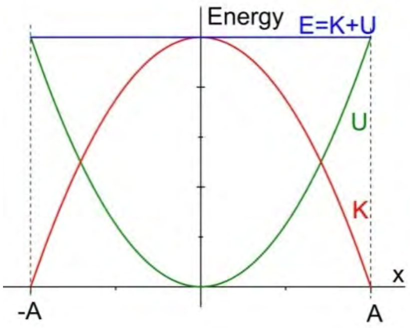
where m is the mass and k is the spring constant.

- Period of a simple pendulum:

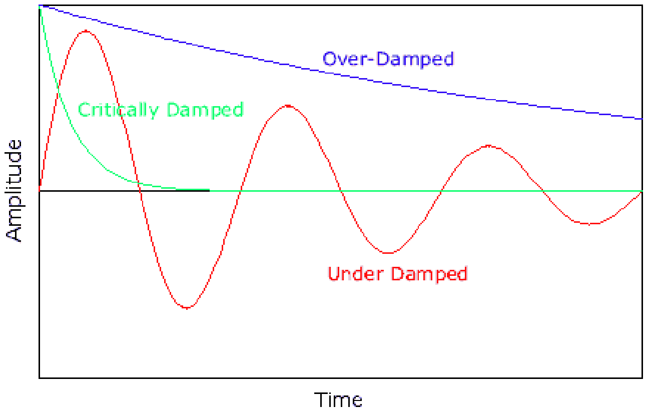
$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is the length of the pendulum and g is the acceleration due to gravity.

- Total of Kinetic Energy (K) and Potential Energy (U) is constant, assuming no energy is dissipated due to friction or air resistance.
    - K is max at the centre (where v is maximum)
    - U is max at maximum displacement (where v is zero)



- Resonance occurs when the driving force matches the natural frequency.
- Damping is the effect that reduces the amplitude of oscillations.
    - Critical damping: System reaches equilibrium as fast as possible without oscillating.
    - Over-damping: System does not oscilate, but reaches equilibrium slowly.
    - Under-damping: System continues oscillating with smaller amplitude and eventually reaches equilibrium.



Gravitation

Newton's Law

- There's an attractive force between all masses

$$F = G\frac{m_1 m_2}{r^2}$$

where G is the gravitational constant and r is the distance between point masses m_1 and m_2

Gravitational Field Strength

- A force field is a region where an object experiences a force.
- Gravitational field lines show the force that a small test mass experiences in such a region.

- Gravitational field strength is the force per unit mass that an object experiences in the field:

$$g = \frac{F}{m}$$

- In a radial gravitational field around mass M,

$$g = \frac{GM}{r^2}$$

Gravitational Potential

- Gravitational potential is the work done in bringing an object with unit mass from infinity to a point in a gravitational field.

$$V = -\frac{GM}{r}$$

- Gravitational potential difference is the difference in the gravitational potentials of two points in a gravitational field.
- Just like gravitational field strength, gravitational potential is a property of a particular point in the gravitational field and does not depend on the object in the field.
- V and g can be related by:

$$g = -\frac{\Delta V}{\Delta r}$$

- The work done in moving an object of mass m between two points in a gravitational field is:

$$\Delta W = m\Delta V$$

Orbits of Planets and Satellites

- The orbiting object (mass m) is in circular motion. Use $F = ma$ with $F = G\frac{Mm}{r^2}$ and $a = \frac{v^2}{r} = \omega^2 r$
- Equation can be solved to find speed (v), angular speed (ω), radius of the orbit (r), or using $T = \frac{2\pi}{\omega}$, its period (T).

- A lower orbit (smaller r) has less potential energy and more kinetic energy than a higher orbit (bigger r).

- Geosynchronus orbits have a period of one day.

Electric Fields

Coulomb's Law

- The force between two point charges in a vacuum is:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

where ε_0 is the permittivity of free space and r is the distance between point charges Q_1 and Q_2

- Force is repulsive between like charges and attractive between unlike charges.

Electric Field Strength

- A force field is a region where an object experiences a force.
- Electric field lines show the force that a small positive test charge experiences in such a region.

- Electric field strength is the force per unit charge that an object experiences in the field:

$$E = \frac{F}{Q}$$

- In a radial electric field around charge Q,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

- In a uniform electric field, where there's a potential difference of V in a distance d,

$$E = \frac{V}{d}$$

Electric Potential

- Electric potential is the work done in bringing a positive unit charge from infinity to a point in an electric field.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

- Electric potential difference is the difference in the electric potentials of two points in an electric field.
- Just like electric field strength, electric potential is a property of a particular point in the electric field and does not depend on the object in the field.
- The work done in moving an object of charge Q between two points in an electric field:

$$\Delta W = Q\Delta V$$

Comparison of Electric and Gravitational Fields

- The concepts of field, field strength and potential are the same for both, 'charge' in electric fields is replaced by 'mass' in gravitational fields
  - Similarity: Both are inverse square laws.
  - Difference: Gravitational force is always attractive, electric force can be both attractive and repulsive

Capacitance

Capacitance

- Capacitance (C) is the charge (Q) stored per unit voltage (V).

$$C = \frac{Q}{V}$$

Energy Stored by a Capacitor

- Energy stored in a capacitor is given by:

$$E = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C}$$

- The energy stored is equal to the area under charge against pd graph.

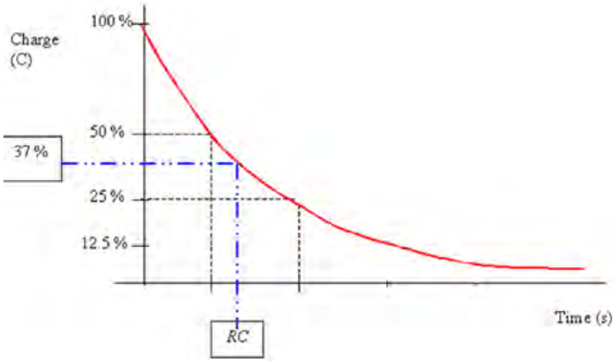
Capacitor Discharge

- Time constant (RC) determines how long it takes a capacitor to charge or discharge.
- After one time constant, the charge in a capacitor falls to 37% of the initial value.
- A capacitor is considered fully discharged after 5 time constants.
- The charge on a discharging capacitor is given by:

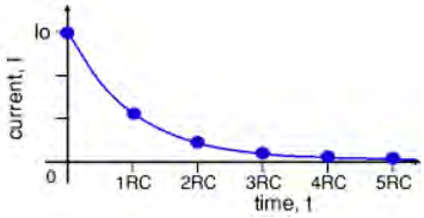
$$Q = Q_0 e^{-t/RC}$$

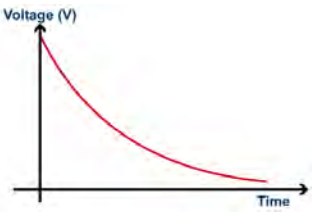
where Q_0 is the initial charge on the capacitor.

- The time constant can be found graphically:

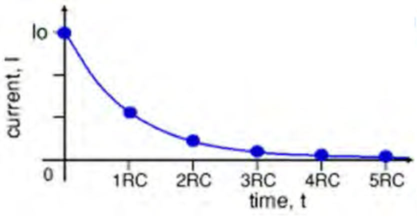


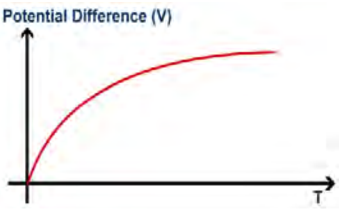
- The graphs for current and pd for a discharging capacitor are similar:

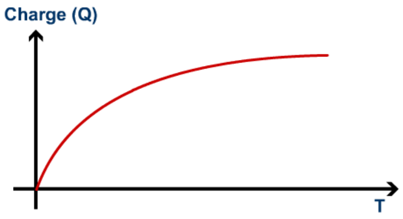




- For a charging capacitor,

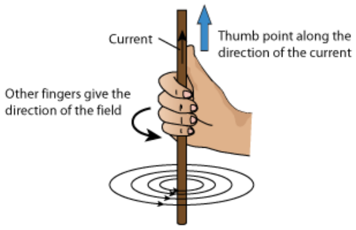






Magnetic Fields

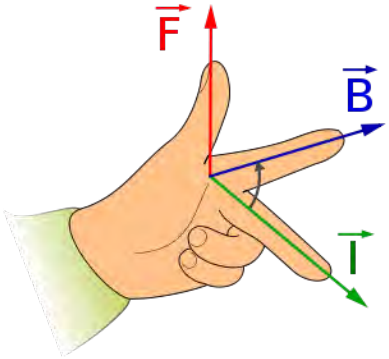
- A magnetic field is formed around a current carrying wire. Use Right Hand Grip Rule:



Right Hand Grip Rule

- A current carrying wire experiences a force in a magnetic field. Use Fleming's Left Hand Rule. The magnitude of the force is:

$$F = BIl$$

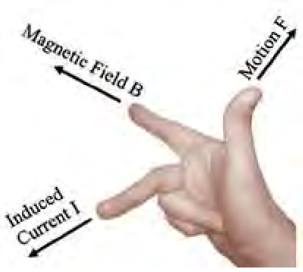


- A charge moving in a magnetic field experiences a force. A moving charge is the same as a current. Use Fleming's Left Hand Rule. The magnitude of the force is given by:

$$F = Bqv$$

  Cyclotrons use magnetic fields to keep charged particles in circular orbits.

- A current is induced in a wire if it moves in a magnetic field. Use Fleming's Right Hand Rule:



- Flux:
  - $Magnetic\ flux\ density = B$
  - $Magnetic\ flux, \phi = BA$
  - $Magnetic\ flux\ linkage = N\phi = BAN$

- Faraday's Law: The induced e.m.f. is directly proportional to the rate of change of magnetic flux linkage
- Lenz's Law: The direction of the induced e.m.f. is such as to oppose the change that induces it.

$$\varepsilon = -N\frac{\Delta\phi}{\Delta t}$$

- E.m.f. induced in a coil in a uniform field:

$$\varepsilon = BAN\omega \sin \omega t$$

- Step-up transformers increase the voltage

$$\frac{N_S}{N_P} = \frac{V_S}{V_P}$$

- High voltage is used in the National Grid to reduce power loss to resistance of cables.
- $Transformer\ efficiency = {}^{I_S V_S}/_{I_P V_P}$
- Causes of inefficiency:
    - Eddy currents
    - Resistance in coils
    - Not all of magnetic flux though coil one passes through coil two